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THE

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AUGUST, 1860.

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J. D. RUNKLE, A.M., A.A.S.

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THE

MATHEMATICAL MONTHLY.

Vol. II. . . . AUGUST, 1860. . . . No. XI.

PRIZE PROBLEMS FOR STUDENTS.

I. Prove that an equation of the second degree cannot have more than two roots.

II. The sum of n numbers in arithmetical progression is a, and the sum of their squares is b. Required, the first term and the common difference. — Communicated by Artemas Martin, Franklin, Pa.

III. Given, three lines, a, b, c, drawn from any point within a regular polygon of n sides to any three of its consecutive corners, A, B, C. Required, the side of the polygon, and also a geometrical solution, granting that a polygon of n sides can be constructed when one side is given. — Communicated by Prof. C. A. Young, Western Reserve College, Hudson, Ohio.

IV. Deduce the formulæ of Arithmetical and Geometrical Progressions by the method of differences. — Communicated by Prof. Henry H. White, Kentucky University, Harrodsburg.

V. If three points on the surface of a sphere (radius $=1$) be joined by three arcs of small circles whose angular radii are $\varrho, \varrho', \varrho''$, the area of the included triangle will be

$$\theta + \theta' + \theta'' - \pi \pm \varphi \cot \varrho \pm \varphi' \cot \varrho' \pm \varphi'' \cot \varrho'';$$

in which $\varphi, \varphi', \varphi''$ are the lengths of these arcs, and $\theta, \theta', \theta''$, the angles at which they intersect. — Communicated by Asher B. Evans.

Solutions of these problems must be received by October 1. 1860.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. VIII., Vol. II.

The first Prize is awarded to G. B. Hicks, Cleveland, Ohio.

The second prize is awarded to Asher B. Evans, Hamilton, N. Y.

The third prize is awarded to J. G. Weinberger, Millersville, Pa.

Prize Solution of Problem I.

By J. G. Weinberger, Normal School, Millersville, Pa.

I. If x is a whole number, prove that $x^3 - x$ is always divisible by 6.

Since $x^3 - x = (x-1)\,x\,(x+1)$, it appears that when x is a whole number, the given expression is the product of three consecutive whole numbers. But we know that at least one of three successive whole numbers is divisible by 2, because at least one must be even; and also, that one of them must be divisible by 3, because if the first one is not divisible there must be a remainder of either 1 or 2; and hence it follows that either the second or third number will be divisible. Therefore $x^3 - x$ is divisible by $2 \cdot 3 = 6$.

Prize Solution of Problem II.

By D. G. Bingham, Ellicotteville, N. Y.

II. Describe a series of circles whose areas shall be one half, one fourth, one eighth, &c. that of a given circle.

Draw from C, the centre of the given circle, two radii, CA and CB, perpendicular to each other, and draw AB, the chord of 90°. From C drop a perpendicular, CK, upon this chord, and with C as a centre, and radius CK, describe a circle; then, as is easily seen,

$$(1) \qquad CA^2 : CK^2 = 2 : 1.$$

But since the areas of two circles are to each other as the squares of their radii, it follows from (1) that the area of the circle CK is half the circle CA. In precisely the same way we can find a circle

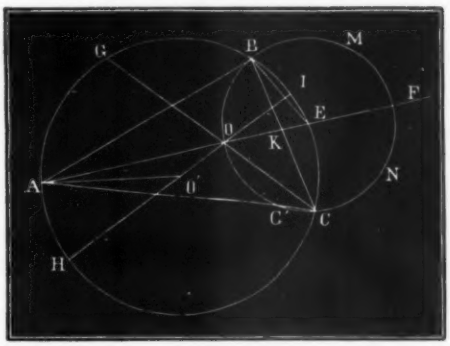
which is half of the circle $C K$, and therefore one fourth of circle $C A$. The process may be continued as far as we please.

Prize Solution of Problem III.

By George B. Hicks, Cleveland, Ohio.

III. If R and r be the radii of the circles circumscribing and inscribing any triangle, and D the distance between their centres, then $D^2 = R^2 - 2Rr$. Required, a geometrical demonstration.

Let O' be the centre of the circumscribed, and O of the inscribed circles. Produce AO indefinitely, and OO' to meet the circumscribed circle in H and I. Draw CG, CE, EB. Lastly, from E as centre, with radius EO, describe the circle $ONFM$. This circle will pass through B and C. For ECO is measured by $\frac{1}{2}(GB + BE)$ and EOC by $\frac{1}{2}(AG + EC)$. But $GB + BE = AG + EC$. Hence the triangle EOC is isosceles, and $EC = OE$. Similarly $BE = OE$. Also, since AE bisects BAC, we have



$$BK = \frac{AB \times BC}{AB + BC}.$$

The chords HI, AE give

$$HO \times OI = (R + D)(R - D) = AO \times OE.$$

But it is shown in Geometry that $(R + D)(R - D) = R^2 - D^2$;

$$\therefore D^2 = R^2 - AO \times OE.$$

It remains to show that $AO \times OE = 2Rr$.

The similar triangles ABK, AEC give $\frac{OE}{AO + OE} = \frac{BK}{AB}$;

(1) $\therefore \frac{OE}{AO+2OE} = \frac{BK}{AB+BK}$, or $\frac{OE}{AF} = \frac{BC}{AB+AC+BC}$.

And from the secants AF, AC, $AO \times AF = AC' \times AC$. But since E is on the bisector of the angle BAC, $AC' = AB$;

(2) $$\therefore AO \times AF = AB \times AC.$$

Multiply (1) and (2), and $AO \times OE = \frac{AB \times AC \times BC}{AB+AC+BC}$. But $r(AB+AC+BC) = \frac{AB \times AC \times BC}{2R}$, since each side of this expression represents double the area of ABC;

$$\therefore \frac{AB \times AC \times BC}{AB+AC+BC} = AO \times OE = 2Rr,$$

as required.

Prize Solution of Problem IV.

By George B. Hicks, Cleveland, Ohio.

IV. If a circle cut a conic section in four points, A, B, C, D, and a second circle cut the same conic section in A, B, E, F, then will CD and EF be parallel.

Let $y^2 - mx - nx^2 = S = 0$ be the equation of any conic, and $x\cos a + y\sin a - p = \alpha = 0$, $x\cos b + y\sin b - p = \beta = 0$ (1), the equations of the chords AB, CD.

Then will $S + k\alpha\beta = 0$ (2) be the equation of *some figure* through the intersections of (1) with the conic, the species of (2) being determined by the value attributed to the arbitrary constant k; for (2) is satisfied by $S = 0, \alpha = 0$; $S = 0, \beta = 0$, and therefore passes through the intersections of S, α, and β. (Salmon's Conic Sections, p. 211.)

To determine k so that (2) may represent a circle, write for S, α, and β their values at full length, equate the coefficients of x^2 and y^2, and equate to zero the cofficient of xy. We get

$$1 + n = k\cos(a+b), \qquad k\sin(a+b) = 0.$$

Squaring and adding,

$$(1 + n)^2 = K^2 \quad \therefore \quad K = n + 1.$$

And we must therefore have

$$\cos(a + b) = 1, \qquad \sin(a + b) = 0,$$

$$\therefore \quad a + b = 0.$$

Now a and b are the angles made with the axis of x by the perpendiculars from the origin on the lines AB, CD.

Hence AB and CD are equally inclined to the axis of the conic (this being the axis of x). So also are AB, EF, since the property just proved is evidently general.

And therefore CD, EF make equal angles with the axis; $\therefore$ they are *parallel*.

Mr. Trowbridge adds to his solution of this problem the following:

"If two conics of the same kind cut a third conic in the points A, B, C, D and A, B, E, F respectively, then will CD and EF be parallel, provided the transverse axes of the cutting conics are either parallel or perpendicular to each other."

Prize Solution of Problem V.

By Asher B. Evans, Madison University, Hamilton, N. Y.

V. In any conic section, let u denote the length of the perpendicular dropped from any point in the curve upon the directrix, r the distance of this point from the nearest focus, a and b the semi-axes; prove that

$$\frac{u^2 - r^2}{u^2} = \frac{b^2}{a^2}.$$

A conic section is the locus of a point whose distances from a given fixed point, called the focus, and a straight line given in position, called the directrix, are always to one another in a constant ratio. This condition gives $r : u :: e : 1$, or $r^2 = u^2 e^2$, where e is the eccentricity of the conic section. Therefore $\frac{u^2 - r^2}{u^2} = 1 - e^2 = \frac{b^2}{a^2}$ for the ellipse, $-\frac{b^2}{a^2}$ for the hyperbola, and zero for the parabola.

NOTES AND QUERIES.

1. *Problems in Division, in which all the figures in the divisor but the right-hand one are nines.* — Let it be required to divide 864835 by 996. Proceed as follows: —

$$\begin{aligned} 864835 = 864 \times 1000 + 835 &= 864\,(996 + 4) + 835, \\ &= 864 \times 996 + 864 \times 4 + 835, \\ &= 864 \times 996 + 3456 + 835. \end{aligned}$$

Hence 996 is contained in 864835, 864 times, and $3456 + 835$ remainder. But

$$3456 = 3\,(996 + 4) + 456 = 3 \times 996 + 3 \times 4 + 456;$$

hence 3456 contains 996, 3 times, with $3 \times 4 + 456 = 468$ remainder. Hence, 864835 contains 996 $864 + 3$ times, with $835 + 468 = 1303$ remainder. But

$$\begin{aligned} 1303 = 1\,(996 + 4) + 303 &= 1 \times 996 + 4 + 303, \\ &= 1 \times 996 + 307; \end{aligned}$$

that is, 1303 contains 996 once, with 307 remainder; and the final result is 868 as the quotient, and 307 remainder.

We can now condense the work into the following

Operation.	Explanation.
864|835 3|456 = 864 × 4 | 12 = 3 × 4 867 = quotient 1|303 = remainder | 4 = 3 × 4 868 = quotient 307 = remainder	Cut off as many figures from the right of the dividend as there are figures in the divisor. Multiply the remaining figures of the dividend, 864, by 4, the complement of the divisor, and set the product underneath the dividend. All the figures of this product over three, the number in the divisor, multiply by 4, and set the product underneath; and so continue to do until none fall to the left of the vertical line. Next add, and all at the left of the vertical line, as 867, will be quo-

tient; and all to the right, as 1307, will be remainder. Treat this remainder in the same way, and so on; and the last additions will give the true quotient and remainder. — Prof. Devolson Wood, University of Michigan, Ann Arbor.

2. *Derivative of* a^x. — Let $u = a^x$; then $\log u = x \log a$. By algebra the Naperian log of u is

$$\log u = \log \frac{z+1}{z} = 2\left(\frac{1}{2z+1} + \frac{1}{3(2z+1)^3} + \frac{1}{5(2z+1)^5} + \&c.\right),$$

$$= 2\left\{\frac{u-1}{u+1} + \tfrac{1}{3}\left(\frac{u-1}{u+1}\right)^3 + \tfrac{1}{5}\left(\frac{u-1}{u+1}\right)^5 + \&c.\right\}$$

since, if $u = \frac{z+1}{z}$, then $\frac{1}{2z+1} = \frac{u-1}{u+1}$.

By differentiating, we get

$$4\,du\left\{\frac{1}{(u+1)^2} + \frac{(u-1)^2}{(u+1)^4} + \frac{(u-1)^4}{(u+1)^6} + \frac{(u-1)^6}{(u+1)^8} + \&c.\right\} = dx \log a.$$

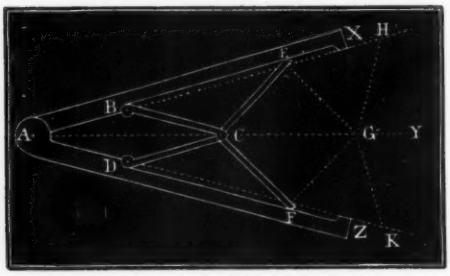
The series in the parenthesis is converging for all positive values of u, and by summing an infinite number of terms the equation becomes

$$4\,du\left(\frac{1}{4u}\right) = dx \log a. \quad \therefore \quad \frac{du}{dx} = a^x \log a.$$

— Lucius Brown, Fall River, Mass.

3. *Remarks upon a (supposed) new Instrument for the Mechanical Trisection of an Angle.* — The *Philosophical Magazine* for April, 1860, contains an account, by Mr. Tate, of an instrument for the mechanical trisection of an angle, which is there described as new. The instrument is simple and elegant, and very convenient for the purpose intended. It will be seen, however, that the construction in question has long been known; for on turning to the 252d page of the second edition of the *Traité Analytique des Sections Coniques*, by De l'Hôpital, first published in Paris in the year 1707, and republished in 1776, we

find the description of an instrument designed for the same object, and which, in its essential parts, is precisely the same as that mentioned above. It is somewhat singular that this should thus have escaped notice, especially as the name of De l'Hôpital is by no means an obscure one, and the problem of the trisection of an angle is one of so great historical celebrity. As the subject is one of some interest, we will give a brief description of the instrument employed by De l'Hôpital. AX and AZ are two rulers united by a joint at A. CB, CD, CE, and CF are four others, all equal, and united together by a pin at C, so as to turn freely. CB and CD are likewise connected with AX and AZ by joints at B and D in such a manner that $AB = BC$, and $AD = DC$, so that $ABCD$ is always a parallelogram, the ends E and F of CE and CF sliding easily upon AX and AZ respectively. Through A and C draw ACY. The $\angle ECY = \angle AEC + \angle EAC$. But



$$AEC = \angle EBC = \angle BAC + \angle BCA = 2\angle EAC.$$

$$\therefore ECY = 3\angle EAC.$$

Similarly, $\angle FCY = 3\angle FAC$. $\therefore \angle ECF = 3\angle EAF$.

Therefore, in order to trisect any angle, we have only to place the instrument in such a position that the sides of the rulers CE and CF coincide with the lines forming the given angle; in which case the straight lines drawn through C, parallel to AX and AZ, will divide it into three equal parts. De l'Hôpital has also shown (as is indicated by the dotted lines in the figure) how, by the addition of four rods, each equal to those meeting at C, and connected to each other

in a similar manner at G, as also to CE, CF, AX, and AZ at the points E, F, H, and K respectively, any angle may be divided into five equal parts, and that the construction may be extended in the same way, so as to effect the division of an angle into any odd number of equal parts. — ARTHUR W. WRIGHT, New Haven, Ct.

4. *Note on the Irreducible Case.* — The following is a convenient method for approximating to a root of an equation of the form $x^3 - ax = b$.

Dividing by x, transposing, &c., we have

$$x = \pm\sqrt{a + \frac{b}{x}}.$$

Therefore

$$x = \pm\sqrt{a + \cfrac{b}{\pm\sqrt{a + \cfrac{b}{\pm\sqrt{a + \cfrac{b}{\&c.}}}}}}$$

Now, in order to find an approximate value of this continued fraction, take a few of its terms, and, beginning with the last, find the square root of $a + b$, divide b by this root, then add a to the quotient, extract the square root again, and continue the same process upward. The values of the radicals, thus successively found (which we may denote by R, R', R'', &c.), will be nearer and nearer approximations to the value of x. Considering all the radicals as positive, we shall thus find a positive root of the equation. It may then be depressed to a quadratic. The work may be somewhat abbreviated by the use of logarithms. In the following solution all the figures are retained.

$$x^3 - 6x = 2.$$

$$x = \pm\sqrt{6 + \cfrac{2}{\pm\sqrt{6 + \cfrac{2}{\&c.}}}}$$

Log. 2		0.301030	
" 8		$\overline{0.903090}$(2	
		0.451545	log. R
		$\overline{1}.849485$	" .7071
" 6.7071		$\overline{0.826535}$(2	
		0.413267	" R'
		$\overline{1}.887763$	" .77226
" 6.77226		$\overline{0.830734}$(2	
		0.415367	" R''
		$\overline{1}.885663$	" .76853
" 6.76853		$\overline{0.830494}$(2	
		0.415247	" $R''' = 2.6016$

Here, after four approximations, a value of x is correctly found to four places of decimals. In the equation $x^3 - 3x = 1$, we shall in the same manner find, after four approximations, $x = 1.8793$, which is also correct to four places of decimals.

An equation of the form $x^3 - ax = -b$ reduces to the form

$$x = \pm\sqrt{a - \frac{b}{\pm\sqrt{a - \frac{b}{\&c.}}}}$$

In this case, if we consider each radical as negative, the second term under each radical will be additive; and the process will be exactly the same as before, and the approximations as rapid. Thus in the equation $x^3 - 7x = -7$, the fourth approximation gives $x = -3.048$, which is correct to every figure.

This method, which is new to the writer, applies to the irreducible case of CARDAN, of which the foregoing equations are examples. It serves to find a value of x "to any required degree of correctness." It will be observed that it is entirely independent of any trial.

It is evident that the approximations will be most rapid when $a^3 > b^2$; a condition which the irreducible case always involves. — Prof. E. W. EVANS, Marietta College, Ohio.

5. *Problems in Imaginary Trigonometry.* — PROBLEM 1. To find an arc whose given sine is greater than radius.

Suppose the radius of the circle to be unity, and let $\sin \varphi = e$, and $e > 1$; we are to obtain a general expression for the value of φ.

For convenience, let $i = \sqrt{-1}$; and observe that

$$(1) \qquad i \cos \varphi = i \sqrt{1 - \sin^2 \varphi} = \sqrt{i^2 + \sin^2 \varphi}.$$

From the differentiation of circular functions, we have

$$d \sin \varphi = \cos \varphi \, d\varphi;$$

whence

$$(2) \qquad d\varphi = \frac{d \sin \varphi}{\cos \varphi} = \frac{i \, d \sin \varphi}{\sqrt{i^2 + \sin^2 \varphi}}.$$

Integrating, and then restoring the value of $\sqrt{i^2 + \sin^2 \varphi}$,

$$(3) \qquad \varphi = i \log [\sin \varphi + i \cos \varphi] + \text{constant}.$$

Now, if e be taken at its inferior limit, 1, φ will be 90°, and $i \log [\sin \varphi + i \cos \varphi]$ will be 0; whence

$$90° = \text{constant};$$

$$(4) \qquad \therefore \varphi = 90° + i \log [\sin \varphi + i \cos \varphi].$$

In order to eliminate $\sin \varphi$ and $\cos \varphi$ from this expression, let ψ be an arc, such that $\sin \varphi = \frac{1}{\cos \psi}$; then, from CHAUVENET'S Trig. Eq. 154, we have

$$(5) \quad \tan (45° + \tfrac{1}{2} \varphi) = \sqrt{\left(\frac{1 + \sin \varphi}{1 - \sin \varphi}\right)} = \sqrt{\left(\frac{\cos \psi + 1}{\cos \psi - 1}\right)} = -i \cot \tfrac{1}{2} \psi;$$

and

$$(6) \quad \tan (45° - \tfrac{1}{2} \varphi) = \sqrt{\left(\frac{1 - \sin \varphi}{1 + \sin \varphi}\right)} = \sqrt{\left(\frac{\cos \psi - 1}{\cos \psi + 1}\right)} = i \tan \tfrac{1}{2} \psi.$$

Subtracting (6) from (5), we have (CHAUVENET'S Trig. Eq. 157.

$$2 \tan \varphi = -i (\tan \tfrac{1}{2} \psi + \cot \tfrac{1}{2} \psi).$$

Substituting in the second member, (CHAUVENET's Trig. Eq. 146 and 147,) and reducing, we have

$$\tan \varphi = -\frac{i}{\sin \psi};$$

whence

$$(7) \qquad i \cos \varphi = \frac{i \sin \varphi}{\tan \varphi} = -\tan \psi;$$

and

$$(8) \quad \sin \varphi + i \cos \varphi = \frac{1}{\cos \psi} - \tan \psi = \frac{1 - \sin \psi}{\cos \psi} = \tan (45° - \tfrac{1}{2} \psi).$$

Whence, by substitution in (4), we obtain

$$(A) \qquad \varphi = 90° + i \log \tan (45° - \tfrac{1}{2} \psi),$$

the general expression sought.

PROBLEM 2. — To find an arc whose cosine is greater than radius.

Let $\cos \varphi' = e = \frac{1}{\sin \psi'}$, e being greater than radius. Then, by the course pursued in the first problem, we shall obtain

$$(1) \qquad \varphi' = -i \log (\cos \varphi' + i \sin \varphi').$$

Eliminating $\cos \varphi'$ and $\sin \varphi'$, we obtain, after the necessary reductions,

$$(B) \qquad \varphi' = -i \log \tan \tfrac{1}{2} \psi'.$$

In order to obtain a construction of equations (A) and (B), disregarding the factor, i, substitute $\cos^{-1} \frac{1}{e} = \psi$, and $\sin^{-1} \frac{1}{e} = \psi'$, and remembering that

$$45° - \tfrac{1}{2} \cos^{-1} \frac{1}{e} = \tfrac{1}{2} \sin^{-1} \frac{1}{e},$$

we have

$$(A') \qquad \varphi = 90° + \log \tan \left(\tfrac{1}{2} \sin^{-1} \frac{1}{e}\right),$$

$$(B') \qquad \varphi' = -\log \tan \left(\tfrac{1}{2} \sin^{-1} \frac{1}{e}\right).$$

Putting $t = \varphi - 90°$, and $t' = \varphi'$, and introducing into the sec-

ond members the modulus of common logarithms, and the value of radius in seconds of arc, we have

$$(a) \qquad t = 474942''.27 \log \tan \left(\tfrac{1}{2} \sin^{-1} \frac{1}{e}\right),$$

$$(b) \qquad t' = -474942''.27 \log \tan \left(\tfrac{1}{2} \sin^{-1} \frac{1}{e}\right),$$

in which the logarithms are common.

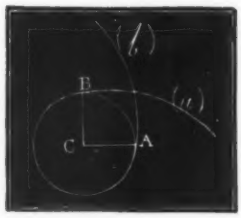
Now observe,

1st. For all values ot e greater than 1, t is essentially negative and t' positive.

2d. If $e = 1$, $t = 0$, and $t' = 0$

3d. If $e = +\infty$, $t = -\infty°$, and $t' = +\infty°$.

Hence, each equation, (a) and (b), is the equation of a *spiral*, of which e is the radius vector, and t, or t', the measuring arc, whose origin is at a distance of unity from the origin of the spiral curve.



The positions of these spirals, relative to the circular arc which corresponds to values of e less than 1, when φ and φ' are real, are readily determined. Let the circular curve have its origin at A, and be reckoned in the direction of AB. Now, since $\varphi = 90°$, and $\varphi' = 0$, when $e = 1$, the origin of the spiral whose equation is (a) is at B, and the origin of the spiral whose equation is (b) is at A; and since t is essentially negative, and t' essentially positive, the directions of the curves will be *backward*, and *forward*, respectively, as shown in the figure.

To give a general idea of the relative increments of t, or t', and e, we add the following results, obtained by computation from (a) and (b).

If $e = 1.005$, $t = -(5° 44' 15'')$, and $t' = 5° 44' 15''$.

If $e = 1.5$, $t = -(55°\ 8'\ 32'')$, and $t' = 55°\ 8'\ 32''$.
If $e = 2.$, $t = -(75°\ 27'\ 21'')$, and $t' = 75°\ 27'\ 21''$.
If $e = 2.508642$, $t = 90°$, and $t' = 90°$.

— Prof. J. C. Porter, Liberal Institute, Clinton, N. Y.

THE MOTIONS OF FLUIDS AND SOLIDS RELATIVE TO THE EARTH'S SURFACE.

[Concluded from Page 346.]

SECTION VII.

ON THE MOTIONS OF SOLID BODIES.*

82. It is proposed in this section to treat of the motions of projectiles and rotating bodies relative to the earth's surface, so far principally as they are influenced by the earth's rotation. By putting P and its derivatives in our fundamental equations equal 0, we get the equations of a projectile. Hence we obtain from equations (9), by restoring the value of α in § 2, the following general equations of the motions of a projectile relative to the earth's surface;—

$$D_t^2 r = r(D_t\theta)^2 + r\sin^2\theta\,(2n + D_t\varphi)\,D_t\varphi - g,$$

$$(61)\qquad r\,D_t^2\theta = -2\,D_t r\,.\,D_t\theta + r\sin\theta\cos\theta\,(2n + D_t\varphi)\,D_t\varphi,$$

$$r\sin\theta\,D_t^2\varphi = -2\sin\theta\,(n + D_t\varphi)\,D_t r - 2r\cos\theta\,(n + D_t\varphi)\,D_t\theta.$$

83. Let α, β, and γ be three rectangular co-ordinates of the projectile, of which α is vertical, β directed toward the south, and γ toward the east, and having their origin at the point of projection.

* Many of the results in this section have been given in a special paper on this part of the subject, published in the fifth volume of Gould's Astronomical Journal. For the sake of completeness, they are here deduced from more general equations and given again.

Let u, v, and w be the lineal velocities respectively in the directions of α, β, and γ, and n' the lineal velocity of the rotation of the earth's surface at the latitude of projection.

Also let R, θ', and φ' be the initial values, or the values at the point of projection, of r, θ, and φ, and u', v', and w' the initial values respectively of u, v, and w.

We shall then have, without any sensible error for the limited range of a projectile,

$$\alpha = r - R, \quad \beta = R(\theta - \theta'), \quad \gamma = R \sin\theta' (\varphi - \varphi'),$$
$$u = D_t r, \quad v = R D_t \theta, \quad w = R \sin\theta' D_t \varphi,$$
$$n' = R n \sin\theta' = \sin\theta' \times 1523.2 \text{ feet}.$$

By means of these equations, equations (61) are reduced to

$$D_t^2 \alpha = \frac{v^2}{R} + \frac{(2n' + w)w}{R} - g,$$
$$D_t^2 \beta = -\frac{2uv}{R} + \frac{(2n' + w)}{R} w \cot\theta', \tag{62}$$
$$D_t^2 \gamma = -\frac{2(n' + w)u}{R} - \frac{2(n' + w)v}{R} \cot\theta'.$$

84. In integrating these equations, g, and also v and w, may be regarded as constant, and equal to their initial values.

By a first integration we get, reckoning t from the time of projection,

$$D_t \alpha = u = u' - gt + \frac{v^2}{R} t + \frac{(2n' + w')w'}{R} t,$$
$$D_t \beta = v = v' - \frac{2u'v'}{R} t + \frac{(2n' + w')w' \cot\theta'}{R} t + \frac{2gv'}{R} t^2, \tag{63}$$
$$D_t \gamma = w = w' - \frac{2(n' + w')u'}{R} t - \frac{2(n' + w')v \cot\theta'}{R} t + \frac{2g(n' + w')}{R} t^2.$$

In integrating the last two equations, the value of $u = u' - gt$, neglecting the other two very small terms, was substituted before integration.

By a second integration we get

$$\alpha = u' t - \tfrac{1}{2} g t^2 + \frac{v'^2}{2R} t^2 + \frac{(2n' + w') w'}{2R} t^2,$$

$$(64) \quad \beta = v' t - \frac{u' v'}{R} t^2 + \frac{(2n' + w') w' \cot\theta'}{2R} t^2 + \frac{2 g v'}{3R} t^3,$$

$$\gamma = w' t - \frac{(n' + w') u'}{R} t^2 - \frac{(n' + w') v' \cot\theta'}{R} t^2 + \frac{2 g (n' + w')}{3R} t^3.$$

These are the complete equations of a projectile in terms of t and its initial velocity. The terms containing n' arise from the earth's rotation, and those having R in the denominator, from the earth's sphericity.

85. If we neglect the terms depending upon the earth's rotation and sphericity in the preceding equations, they may be reduced to

$$(65) \qquad \begin{aligned} \alpha &= u' t - \tfrac{1}{2} g t^2, \\ s &= i t, \end{aligned}$$

in which s is the horizontal co-ordinate, having for its direction the initial direction of the projection, and i the horizontal velocity of projection. Hence the projectile in this case moves in a vertical plane. These are the equations of a projectile as given in elementary treatises, in which no account is taken of the earth's rotation, and in which the directions of gravity are supposed to be parallel. Eliminating t, we get the equation of a parabola, and hence, in this case, the motion is parabolic.

86. In the more general case, the terms containing n' and R in equations (64) deflect the motion from the parabola, and also from the vertical plane of projection. These terms, however, are very small, and their effects may be regarded merely as small perturbations from parabolic motion.

87. When the initial motion of projection is in a vertical direction, $v' = 0$, and $w' = 0$, and equations (64) become in this case

$$\alpha = u\,t - \tfrac{1}{2} g\,t^2,$$

(66) $$\beta = 0,$$

$$\gamma = -\frac{n'u'}{R}\,t^2 + \frac{2\,g\,n'}{3\,R}\,t^3.$$

Hence, since $\beta = 0$, an ascending or falling body projected vertically does not deviate to the north or south of a perpendicular; but the value of γ gives the deviation of such a body to the east or west, and depends entirely upon the earth's rotation.

If a body is projected vertically upward, when it arrives at its maximum height, $u = 0$, and the first of equations (63) then gives, neglecting the very small terms, $u' = g\,t$. Hence the last of the preceding equations gives

(67) $$\gamma = -\frac{u^3 n'}{3\,R\,g^2} = -\frac{g\,n'}{3\,R}\,t^3$$

for the deviation west when the body is at its greatest height.

If a body is let fall from a state of rest relative to the earth's surface, $u' = 0$, and we get in this case

(68) $$\gamma = \frac{2\,g\,n'}{3\,R}\,t^3$$

for the deviation of a falling body east of the perpendicular.

If a body is projected vertically upward, the last of equations (63) gives, when the body is at its maximum height, putting t' for the time of ascent,

$$w = -\frac{2\,n'u'}{R}\,t' + \frac{2\,g\,n'}{R}\,t'^2.$$

In applying the equation to the same body in falling, this value of w becomes the initial velocity, and must be substituted for w'. Hence we shall have

$$D_t\gamma = -\frac{2\,n'u'}{R}\,t' + \frac{2\,g\,n'}{R}\,t'^2 - \frac{2\,n'u'}{R}\,t + \frac{2\,g\,n'}{R}\,t^2.$$

Integrating,

$$\gamma = -\frac{2n'w'}{R}t't + \frac{2gn'}{R}t^2t - \frac{n'w'}{R}t^2 + \frac{2gn'}{3R}t^3.$$

When the body returns to the level from which it is projected, $t = t'$, and $gt = w'$, by which the preceding equation is reduced to

$$\gamma = -\frac{w'^3 n'}{3Rg^2}.$$

It deviates to the west in ascending, we have seen, by the same amount; and hence, if a body is projected vertically upward and returns, it falls to the west of the point of projection a distance equal to twice the preceding value of γ.

All the preceding deviations are small, amounting to only a few inches in a range of several hundred feet. Since the value of n' contains $\sin\theta$ as a factor, they are greatest at the equator and decrease as the sine of the polar distance.

88. When the projectile has a considerable horizontal range, the amount of deflection arising from the earth's rotation is much greater. If the earth had no rotation, the projectile would move in the same vertical plane. The deflection, therefore, from this plane depends upon the terms in equations (64) containing n'. Hence, if $\delta\beta$ and $\delta\gamma$ denote the effect of these terms upon the horizontal co-ordinates, we shall have very nearly, when the projectile does not have a great range in altitude,

$$\delta\beta = \frac{n'w'\cot\theta'}{R}t^2 = nw'\cos\theta' \cdot t^2,$$

(69)

$$\delta\gamma = -\frac{n'v'\cot\theta'}{R}t^2 = -nv'\cos\theta \cdot t^2.$$

The effect of the other terms containing n', we have seen, is very small, unless the range in altitude is great.

Let s be a horizontal co-ordinate in the direction of the horizontal range, and p a co-ordinate at right angles on the right, and i the

horizontal velocity of projection. Then, by resolving the preceding co-ordinates and velocities into the directions of s and p, we obtain

$$p = n i \cos\theta' t^2. \tag{70}$$

If a cannon-ball were to fly 3 miles in 12 seconds, the preceding equation, using the value of n in § 18, would give, at the parallel of 45°, about 10 feet for the value of p, or the deviation to the right of the initial direction.

89. If a body be supposed to move on the surface of the earth without friction, it would be continually deflected into a curve by the force $D_p F$, equation (53). If ρ be the radius of curvature, and m the angular velocity about the centre of curvature, the centrifugal force will be ρm^2, which must be put equal $D_p F$. Hence we have from equation (53)

$$\rho m^2 = 2 n v \cos\theta.$$

Also, since v is the lineal velocity of the moving body,

$$\rho m = v.$$

Hence we have

$$\rho = \frac{v}{2 n \cos\theta}, \tag{71}$$

$$m = 2 n \cos\theta.$$

When the range of motion is so small that $\cos\theta$ may be regarded as constant, ρ and m are constant, and hence the body then moves in the circumference of a circle with a uniform angular velocity. If we put τ for the time of a revolution, we shall have

$$\tau = \frac{2\pi}{m} = \frac{2\pi}{n \cos\theta} = \sec\theta \times 1 \text{ day}. \tag{72}$$

Hence the time of a revolution is independent of the initial velocity of the body.

90. The gradual gyration of a vibrating pendulum is caused by this same deflecting force, and hence the time of gyration is the same as that of τ in the preceding equation.

91. If a body is forced to move in a straight line, the last form of the value of $D_p F$, equation (53), serves to compare the lateral pressure of this body with gravity. If we put $v = 60$, which is a velocity of about 40 miles per hour, the equation gives $D_p F = \frac{1}{578} g$, at the parallel of 45°. Hence if a railroad train moves in a straight line 40 miles per hour at the parallel of 45°, the lateral pressure on the rails is $\frac{1}{578}$ of its weight, and this is precisely the same for all directions, and not for the direction of the meridian only, as has been generally supposed.

92. We shall now examine the effect of the earth's rotation upon a rotating body of revolution, so suspended as to be free to turn about any axis of revolution passing through its centre of gravity. The same deflecting forces arising from the earth's rotation which act upon a free body having a motion relative to the earth's surface, must also act upon the different parts of a rotating body; and unless the sum of the moments of these forces with regard to any axis is equal to 0, it must have a tendency to turn the body around that axis, and consequently change the direction of the axis of rotation, unless it coincides with the axis of the moments.

93. Let A, B, and C be the sums of the moments which tend to turn the rotating body about the axes which are respectively perpendicular to the plane of the meridian, the prime vertical, and the horizon.

Also let r', θ', and φ' be the values of r, θ, and φ belonging to the centre of gravity of the rotating body.

We shall then have

$$
\begin{aligned}
A &= \int_m r\,(r - r')\,D_t^2\,\theta - \int_m r\,(\theta - \theta')\,D_t^2\,r, \\
B &= \int_m r \sin\theta\,(r - r')\,D_t^2\,\varphi - \int_m r \sin\theta\,(\varphi - \varphi')\,D_t^2\,r, \qquad (73) \\
C &= \int_m r^2 \sin\theta\,(\varphi - \varphi')\,D_t^2\,\theta - \int_m r^2 \sin\theta\,(\theta - \theta')\,D_t^2\,\varphi.
\end{aligned}
$$

Let ρ be the distance of any particle of the rotating body from the axis of rotation; α, the distance from the centre of gravity of the plane passing through the particle perpendicular to the axis of rotation; φ, the angle between the plane of the meridian and a vertical plane passing through the axis of the rotating body; ψ, the angle of elevation of the axis above the horizon; i, the angular velocity of rotation; $it + \mu$, the angle between any particle and the vertical plane passing through the axis of rotation. We shall then have

$$
\begin{aligned}
r - r' &= -\alpha \sin\psi + \rho \cos\psi \cos(it + \mu), \\
r(\theta + \theta') &= \alpha \cos\psi \cos\varphi + \rho \sin\varphi \sin(it + \mu) + \rho \sin\psi \cos\varphi \cos(it + \mu), \\
r \sin\theta (\varphi + \varphi') &= -\alpha \cos\psi \sin\varphi + \rho \cos\varphi \sin(it + \mu) - \rho \sin\psi \sin\varphi \cos(it + \mu).
\end{aligned} \tag{74}
$$

In taking the derivatives of r, θ, and φ with regard to t, since φ and ψ change very slowly in comparison with $(it + \mu)$, $D_t\varphi$ and $D_t\psi$ may be neglected in comparison with $D_t(it + \mu) = i$. Also r in the last two equations may be regarded as constant. Hence we get

$$
\begin{aligned}
D_t r &= -i\rho \cos\psi \sin(it + \mu), \\
r D_t\theta &= i\rho \sin\varphi \cos(it + \mu) - i\rho \sin\psi \cos\varphi \sin(it + \mu), \\
r \sin\theta D_t\varphi &= i\rho \cos\varphi \cos(it + \mu) + i\rho \sin\psi \sin\varphi \sin(it + \mu).
\end{aligned}
$$

By substituting these values in the right-hand members of equations (61), and then substituting the values of the first members thus obtained, and also the values of the first members of equations (74), in equations (73), and integrating, we get

$$
\begin{aligned}
A &= -ni(\sin\theta \sin\psi - \cos\theta \cos\psi) \int_m \rho^2, \\
B &= -ni \cos\theta \cos\psi \sin\varphi \int_m \rho^2, \\
C &= -ni \sin\theta \cos\psi \sin\varphi \int_m \rho^2.
\end{aligned} \tag{75}
$$

94. If in these equations we put $\varphi = 0$, and $\psi =$ the complement of θ, which is making the axis of the rotating body parallel with the earth's axis, we get $A = 0$, $B = 0$, and $C = 0$; and hence the axis of the rotating body in this position has no tendency to change its direction. In all other positions the small deflecting forces tend to change its direction.

95. If the rotating body is free to turn about any axis, the axis of rotation moves at right angles to the resultant of the forces which tend to change its direction, upon the same principle that the axis of an ordinary gyroscope gyrates horizontally at right angles to the direction of gravity. Hence it must gyrate around the position which it would have, if parallel with the earth's axis. If the body is free to turn about the axis only perpendicular to the meridian, its motion is oscillatory and determined by the first of equations (75). If it is free to turn about the axis only perpendicular to the prime vertical, its motions are determined by the second of equations (74); and if free to turn about the axis only perpendicular to the horizon, by the last of those equations; in both of which cases the motion is also oscillatory. These forces, however, are so small, even with the most rapid velocity of gyration, and in any experiments with the most delicate apparatus the resistances in comparison are so great, that the preceding motions are not observed, but only a tendency of the axis of the rotating body to assume the position in which all the forces are in equilibrium, which in the general case in which the axis is free to move in any direction, is a position parallel with the axis of the earth.

96. These deductions from theory are in exact accordance with some very delicate experiments made by FOUCAULT with a peculiar form of gyroscope, an account of which is given in the *American Journal of Science and Arts*, Second Series, Vol. XV. p. 263. See also Vol. XIX. p. 141.

SECTION VIII.

CONCLUSION.

97. In the first two sections of the preceding pages we endeavored to determine analytically the motions relative to the earth's surface which would satisfy all the conditions of a fluid surrounding the earth with a rotation on its axis, and also the figure which such a fluid must assume, on the hypothesis that the statical equilibrium of the fluid were very slightly disturbed by a difference of density between the equator and the poles. In the third section we proceeded in a similar manner with regard to a small circular portion of such a fluid, supposing that its statical equilibrium is disturbed by a difference of density between the centre and the external part. All the results obtained are on the hypothesis that the motions of the fluid are not resisted by the earth's surface, or, in the case of a circular portion of the fluid, by the resistance of the surrounding part. In the next three sections the results thus obtained were used to explain the general motions of the atmosphere and the ocean, the variation of barometric pressure at the earth's surface in different latitudes, the motions of cyclones, and the oscillations of the barometer, allowance being made for the modifying effects of friction or resistance from the earth's surface. In the last section the equations for the motions of solids were deduced from the general equations for the motions of fluids, and applied to the determination of the effect of the earth's rotation on the motions of a projectile and of a rotating body at the earth's surface.

98. For the sake of those who are not disposed to enter into a thorough analytical investigation of the subject, we propose to give in conclusion, without regard to quantitative results, a brief ex-

planation of the principal results at which we have arrived analytically, based upon well-known principles, and also to give a few additional items upon several parts of the subject.

99. First, with regard to the general motions of the atmosphere and the ocean, it is well understood that if one part of a fluid has a greater density than another, the difference of pressure causes a current at the bottom from the denser to the rarer part, and a countercurrent at the top. Hence, if a fluid surrounding the earth is less dense in the equatorial than the polar regions, there must be a current at the earth's surface toward the equator, and a current above toward the poles, unless these motions are modified by other forces. If the earth had no rotation on its axis, this interchanging motion would be always in the direction of the meridians, and consequently there would be no motion of the fluid relative to the earth's surface, east or west. But the earth having a rotation on its axis, by the well-known principle of the preservation of areas, the part of the fluid moving toward the poles, since it becomes nearer the axis of rotation, must acquire a greater angular velocity of rotation than the earth, that is, it must acquire an eastward motion relative to the earth's surface. For the same reason the part moving toward the equator, since it becomes farther from the earth's axis, must acquire a westward motion relative to the earth. Hence it is evident that the atmosphere and the ocean, being less dense at the equator than at the poles, on account of a difference of temperature, must have an eastward motion toward the poles, and a westward motion in the equatorial regions. The velocity of this motion, on the hypothesis that it is not resisted by the earth's surface, and that its initial state was that of rest relative to the earth, is determined by equation (31), and the modifying effect of the resistance of the earth's surface has been treated in the fourth section.

100. If every part of a homogeneous fluid surrounding the earth

had the same angular motion of rotation with the earth, it would be in a state of statical equilibrium relative to the earth, having the same elliptical figure or outline, and its pressure upon the earth's surface would be everywhere very nearly the same. The part of the centrifugal force arising from the earth's rotation resolved in the direction of the meridians, which would be necessary to keep it in this state, is evidently $r n^2 \sin\theta \cos\theta$. If now any part of the fluid had a greater angular velocity of rotation expressed by $n + D_t \varphi$, this force would become $r(n + D_t \varphi)^2 \sin\theta \cos\theta$, and the fluid would press toward the equator with a force represented by the difference of these two expressions, which is $r(2n + D_t \varphi) \sin\theta \cos\theta \; D_t \varphi$. If $D_t \varphi$ were negative, unless it were greater than $2n$, this expression would be negative, and the pressure would be toward the pole. Now since the motion of the atmosphere toward the poles is eastward, and near the equator toward the west, it is evident that this pressure is exerted both from the poles and the equator toward the parallel where the atmosphere has no motion east or west, and consequently must cause an accumulation of the atmosphere there and a greater barometric pressure. The figure which the atmosphere must assume in consequence of these pressures is represented by Fig. 5.

101. From what precedes, if a body moves on the surface of the earth in the direction of the meridians, it is deflected to the right in the northern hemisphere, and the same is true if it moves in the direction of the parallels of latitude. Now since in any direction in which a body moves its motion must be compounded of these two motions, the resultant of the two deflecting forces belonging to each of these components must cause the body to be deflected to the right, in whatever direction it may move. In the southern hemisphere the deflection must be to the left. This result has been obtained analytically in § 32, and the amount of this deflecting force is given in equation (53).

102. By the preceding principle we may also explain the motion of a cyclone from the equator toward the poles, independently of our analytic results in § 31. The motion of the equatorial side of a cyclone in either hemisphere is always toward the east, and hence the deflecting force causes a pressure toward the equator, but that of the polar side being always toward the west, the deflecting force causes a pressure toward the pole. Now these deflecting forces being as the sine of the latitude, as may be seen from § 100, the pressure on the polar side toward the pole is greater than that on the other side toward the equator, and hence the cyclone moves in the direction of the greatest pressure. It is not to be supposed, however, that there is an actual transfer of all the atmosphere of a cyclone from the equator to the polar regions. For the motions and pressure of the cyclone being greater on the polar side, where the deflecting forces which cause it are greatest, its action upon the atmosphere in advance of it is greater than on the equatorial side, where these forces are much less, and hence new portions of the atmosphere are being continually brought into action on the one side, while the resistance of the earth's surface, and the adjacent portions of atmosphere on the other side, are continually overcoming the comparatively weak forces there, and destroying the gyratory motion of the cyclone; so that the centre of the cyclone is being continually formed in advanced portions of the atmosphere. Since many cyclones are more than one thousand miles in diameter, the difference in the violence of its action on the two sides is very considerable.

103. This same principle may be used to explain some of the motions of a rotating body. Suppose such a body be placed with its axis of rotation in any direction parallel with the horizon. Then the motions of the upper and the lower parts being in contrary directions relative to the earth, and the deflecting force in both be-

ing either to the right or the left, according to the hemisphere in which it is, gives the axis of rotation a tendency to assume a perpendicular position. But there are other forces beside these horizontally deflecting forces, so that all the forces which tend to change the position of the axis would not be in equilibrium with the axis in that position. For the equatorial side then would have a motion coinciding in direction with the motion of the earth's rotation, while the other side would have a motion the contrary way, and consequently the centrifugal force arising from the motion of the earth's rotation, combined with that of the rotating body, would be greater on the equatorial than on the polar side, and give the axis of the rotating body a tendency to move in the plane of the meridian. It might be easily shown, when the axis has a position parallel with the axis of the earth, that the forces which tend to change its direction are then in equilibrium, and consequently the axis, if free to turn in any direction, does not change its position.

104. We shall now give a little additional matter upon several points which would more properly have come in the preceding sections. It has been seen (§ 100), that, in consequence of the earth's rotation, the interchanging motion of the atmosphere between the equator and the poles gives rise to a force, represented by $r(2n + D_t\varphi) \sin\theta \cos\theta \, D_t\varphi$, by which this motion itself is counteracted. For instance, the motion toward the poles in the upper regions causes an eastward motion which gives rise to a force toward the equator, and which, consequently, counteracts the motion toward the poles, and the motion toward the equator produces a westward motion which gives rise to a force acting in the direction of the poles, which counteracts the motion toward the equator. The motion of the atmosphere, therefore, between the equator and the poles, is not produced by the whole force arising from the difference of density between the equator and the poles, but by a small dif-

ference only between the two forces. The difference of the preceding forces has been represented by W (§ 44), and it was shown in § 49 that it must in general be very small in comparison with those forces themselves. Hence if the earth had no rotatory motion, the force which produces this motion would be very much greater, and there would be a sweeping hurricane from the pole to the equator.

105. The approximate east or west motion of the atmosphere at the earth's surface, and also at the height of three miles, was computed for several parallels of latitude, from equation (60), neglecting the term W, and given in the table in § 48. The computation is based upon small differences of barometric pressure, and also upon the hypothesis that the difference of temperature between the equator and the poles is 60°; and was designed principally to give a general idea of what the motions must be in the upper regions, where the error arising from neglecting W must be the least. The atmosphere is extremely mobile, and consequently is very much disturbed by a great many local and temporary causes, which our investigation does not take into account, so that it is only the average velocity of a very changeable motion which can be compared with our results. There is no determination of such an average for the upper regions of the atmosphere, and even at the earth's surface this average, from the nature of the observations, cannot be accurately determined. It is, however, known from observations of the motions of the clouds, and also of balloons, that the eastward motion is much greater above than at the earth's surface, in accordance with our computed results. In Professor COFFIN'S Treatise on the Winds (*Smithsonian Contributions*, Vol. VII. p. 184) is a table of resultants determined from observations on the velocity and direction of the wind, from which the average east or west velocity may be determined for comparison with our computed results for the earth's surface. The average of all the observations in England and Scot-

land gives an eastward velocity of about six miles per hour, and the average of all in the Middle and Western United States gives an eastward velocity only a little more than two miles per hour. The computed velocity, according to our table, for the average latitude of the former observations, is about nine miles per hour, and for that of the latter, about four miles. Hence the computed results are from two to three miles less than those determined by observation, which is evidently the effect of the term W in equation (60), which was neglected. For since the atmosphere has a motion toward the poles in the middle latitudes, W is there positive (§ 49), and hence the neglect of it diminishes the results. This difference between the observed and computed results furnishes the value of W for those latitudes at the surface of the earth, and since W must be greatest there, the omission of it cannot cause an error of the same amount in the computed results for the upper regions.

106. With regard to the gyratory motion of the oceans, it may be further added here, that such gyrations are clearly demonstrated by the positions of the isothermal lines, as has been shown by Professor DANA, in a paper read at the twelfth meeting of the American Association for the Advancement of Science (*Proceedings*, Vol. XII. p. 77). According to this paper, the isothermal line of 68° F., in winter, extends, in the North Atlantic, from 56° N. on the American side, to 12° N. on the African, and in the South Atlantic, from latitude 31° S. on the South American coast, to 7° S. on the African side. Similar evidences are given of gyratory motions, in a less degree, in both the North and South Pacific, and also in the Indian Ocean.

107. The cause of the maximum accumulation of the atmosphere near the parallel of 30°, and the explanation of the southwest winds in the middle latitudes, which are produced by it, were first given in my former essay, published in the year 1856. At the meeting of the British Association for the Advancement of Science, in the year

1857, Mr. THOMSON read a short paper in which he explains this accumulation of atmosphere, and the consequent reversion of the lower strata of the atmosphere in the middle latitudes, as arising from the centrifugal force of the eastward motion of the atmosphere, and illustrates the effect of such a force by means of a gyrating vessel of water in which the surface water recedes from the centre, while at the bottom there is a flowing toward it. In this paper nothing is said of the influence of the earth's rotation, and if he means that the effect is produced simply by the centrifugal force arising from the eastward motion of the atmosphere relative to the earth's surface, independent of the earth's rotation, the force would not be great enough to produce any sensible effect. For by examining the expression of the force which produces this effect, in § 100, it is seen that it depends principally upon the earth's rotation, since $D_t \varphi$, the angular velocity of the atmosphere relative to the earth, is small in comparison with $2n$, which is double the velocity of the earth's rotation. That the motion of the lower strata only in the middle latitudes is reversed, by this cause, as Mr. THOMSON states, and as is represented by Fig. 5 in this paper, and not the motion of the whole atmosphere in those latitudes, as has been supposed, and as was represented in my first essay, is undoubtedly true; but instead of only a thin stratum next the earth's surface, as Mr. THOMSON thinks, observations on the motions of the clouds indicate that this stratum extends to a considerable height.

The reader, who has also read my former essay, will notice one or two other slight changes in the results, but all the main points, which were imperfectly set forth in that essay, are fully established by the present more thorough investigation of the subject.

CAMBRIDGE, Mass., February, 1860.

Editorial Items.

We have received the following solutions of the Prize Problems in the May number of the Monthly: —

Harriet S. Hazeltine, Worcester, Mass., Probs. I., II.
J. G. Weinberger, Pennsylvania State Normal School, Millersville, Prob. I.
George B. Hicks, Cleveland, Ohio, Probs. III., IV., V.
Asher B. Evans, Madison University, N. Y., Probs. I., II., III., IV., V.
David Trowbridge, Perry City, N. Y., Probs. I., II., IV., V.
D. G. Bingham, Ellicotteville, N. Y., Prob. II.
Gustavus Frankenstein, Springfield, Ohio, Prob. IV.

Mathematical Monthly Notices.

Translation of the Sûrya-Siddhânta, A Text-Book of Hindu Astronomy, with Notes and Tables, and an Appendix containing additional Notes and Tables, Calculations of Eclipses, a Stellar Map, and Indexes. From the Journal of the American Oriental Society, Vol. VI. New Haven, 1860. 8vo. pp. 364. Sold by the Society's Agent, John Wiley, 56 Walker Street, New York.

The astronomy of the Hindus, though discussed at length by Bentley, Delambre, Biot, and various others, has never before been presented to us in the only satisfactory form, by a translation of one of their own treatises. We have had no means of knowing the real extent and character of Hindu science, or of comparing it with that of other nations to learn its origin. The present work furnishes that information, and is a very important contribution to the history of the exact sciences, — by far the most important ever published in this country. The first rough draft of the translation, as we learn from the preface, was made in India by Rev. E. Burgess, formerly Missionary of the A. B. C. F. M., aided by a native astronomer. The whole collected material was however placed in the hands of the committee of publication of the Oriental Society for revision, expansion, and reduction to a suitable form. It is not difficult to see in every line the careful, conscientious scholarship of Professor Whitney, and the lovers of the exact sciences owe to him the greater debt, that he, a philologist, has made so rich a contribution to their favorite department of knowledge.

The Sûrya-Siddhânta is one of the earliest and most esteemed of the many text-books of Hindu Astronomy. We are by no means, however, compelled to believe in the Hindu date, which makes it a revelation from the *Sun* given about 2,163,101 B. C. There are, however, indications of its existence as early as the sixth century of our era, and the astronomical data correspond best with even an earlier epoch.

The treatise is poetical, and consists of very brief elliptical rules for calculating the places of the heavenly bodies, solar and lunar eclipses, and other celestial phenomena. These rules would be wholly unintelligible without a teacher or commentary, or some knowledge beforehand of what should be said. The whole system presents a strange mixture of very exact knowledge and the most arbitrary and fanciful assumptions. The planetary theory is essentially that of the Greeks, given by Ptolemy in the Syntaxis, differing from it only in some minor particulars. In each the system of epicycles is fundamental. A full and careful comparison with the Syntaxis

is presented in the notes. It is only by such means that we can decide how far the Hindus were indebted to others for their astronomy. To show the accuracy of some of their determinations we quote (from p. 24) the following table of the times of sidereal revolutions, as given by different authorities.

Planet.	Sûrya-Siddhânta.	Siddhânta-Çiromaṇi.	Ptolemy.	Moderns.
	d. h. m. s.	d. h. m. s.	d. h. m. s.	d. h. m. s.
Sun, ...	365 6 12 36.56	365 6 12 9	365 6 9 48.6	365 6 9 10.75
Mercury, . .	87 23 16 22.26	87 23 16 41.52	87 23 16 42.9	87 23 15 43.89
Venus, . . .	224 16 45 56.19	224 16 45 1.93	224 16 51 56.8	224 16 49 7.99
Mars,. . . .	686 23 56 23.48	686 23 57 1.50	686 23 31 56.1	686 23 30 41.41
Jupiter, . . .	4,332 7 41 44.33	4,332 5 45 43.69	4,332 18 9 10.5	4,332 14 2 8.55
Saturn, . . .	10,765 18 33 13.25	10,765 19 33 56.48	10,758 17 48 14.9	10,759 5 16 32.22
Moon:				
Sid. revolut'n,	27 7 43 12.63	27 7 43 12.12	27 7 43 12.1	27 7 43 11.42
Synod. rev., .	29 12 44 2.80	29 12 44 2.28	29 12 44 3.3	29 12 44 2.89
Rev. of apsis,	3,232 2 14 53.43	3,232 17 37 5.98	3,232 9 52 13.6	3,232 13 48 29.64
" " node,	6,794 9 35 45.40	6,792 6 5 41.88	6,799 23 18 39.4	6,798 6 41 45.60

As a contrast to this exact knowledge, it may be mentioned that the positions of twenty-eight fixed stars are given, but eight of them have errors of over three degrees either in latitude or in longitude. We mention again, as characteristic of the system, that the nodes and apsides of all the planets are made to revolve, but with so slow a motion withal that even with the best modern instruments, it could not in some cases be detected in hundreds of years. We commend the note on pp. 26 – 28 to the attention of the curious.

This mixture of exact knowledge, gross error, and arbitrary assumption suggests the conclusion, which the arguments of Prof. Whitney prove, that the science of the Hindus was derived from the Greeks, being worked over by the borrowers to suit their fanciful mythology and chronology, and to serve as a book of rules for the astrologers and for the calculation of eclipses.

The origin of the Hindu division of the zodiac into twenty-seven or twenty-eight parts, and its connection with the Chinese zodiac and the Arabic lunar mansions is very fully discussed, and illustrated by a stellar map, and much light is thrown on this obscure subject.

The amount of the knowledge of the Hindus in arithmetic, geometry, and trigonometry are indicated by this treatise. The decimal notation and the sine of an arc were known. Angles are not mentioned in the treatise. A love of calculation is quite prominent. Multiplications and divisions, where the numbers have ten or twelve figures each, are quite frequent. Almost the only geometrical knowledge involved in the rules is that of the forty-seventh problem of Euclid and that of the proportionality of the sides of similar right-angled triangles. Of these, however, they make constant and dexterous use, solving, in fact, all ordinary cases of plane and spherical trigonometry.

The notes are quite full, being designed to satisfy the wants of two classes, the philologists who are not astronomers, and astronomers who are not philologists. We heartily commend this work to the attention of those who are interested in the history of astronomy or mathematics.

There are a great many other treatises of Hindu astronomy, and some of them are of considerable age. A comparison of these treatises would doubtless serve to show what points are peculiar to the Hindu astronomy, and what were borrowed from other systems. We sincerely hope that Prof. Whitney, who has shown himself so admirably qualified for the task, will place science under renewed obligations by undertaking it.

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